fusion tuition

O-LEVEL 2026 · ADDITIONAL MATHEMATICS · 4049/01

2026 A-Math Paper 1

worked solutions

Every question, step by step, with the method (M), accuracy (A) and independent (B) marks shown where examiners would award them. Mark your own script as you go.

SAT ON	QUESTIONS	TOTAL	DURATION
26 Oct 2026	12	90 marks	2 h 15 min

Your total ____ / 90

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BEFORE YOU START

How to use these solutions

How marks are shown

M1 **Method mark.** For a correct method applied to the question, even if an arithmetic slip follows.

A1 **Accuracy mark.** For a correct result. It depends on the method mark before it.

B1 **Independent mark.** For a correct statement or value, whatever the method.

How to mark yourself: compare each line of your working with the steps, tick the marks you earned, and write your score in the box at the top of each question. The "Will I still get the mark?" section covers the alternatives students ask about most.

Accuracy: give non-exact answers to 3 significant figures, and angles in degrees to 1 decimal place, unless the question says otherwise.

Contents

1	Differentiation and integration	4 marks
2	Surds (no calculator)	6 marks
3	Discriminant: line meets curve	6 marks
4	Exponential growth with logarithms	7 marks
5	Line meets curve: simultaneous equations	6 marks
6	Completing the square	8 marks
7	Equation of a circle	7 marks
8	Maximum volume (differentiation)	8 marks
9	Trigonometric identity, hence solve	10 marks
10	Area under a trigonometric curve	10 marks
11	Coordinate geometry: trapezium and symmetry	10 marks
12	Stationary points of inflexion	8 marks

QUICK CHECK

Answers at a glance

Q	PART	ANSWER	MARKS	YOURS
1	(a)	$2x\sin x + x^2\cos x$	2	
	(b)	$\frac{3}{2}\ln(2x+1)+c$	2	
2	(a)(i)	$k = 8$	2	
	(a)(ii)	$x = \dfrac{-5-\sqrt{57}}{2}$	1	
	(b)	$6+3\sqrt{2}$	3	
3		$k < -\frac{2}{13}$ or $k > 10$	6	
4	(a)	Level 21	3	
	(b)	Level 14	4	
5		$(1,-1)$ and $(6,9)$	6	
6	(a)	$A = 5,\ B = -1,\ C = k-5$	3	
	(b)(i)	$(1,\ k-5)$	2	
	(b)(ii)	$y \geqslant k-5$ for all x	1	
	(b)(iii)	$k \geqslant 8$	2	
7	(a)	$(x-2)^2+(y-3)^2 = 58$	3	
	(b)	$p = 4,\ q = 6,\ r = 6$	4	
8	(a)	$V = 40\pi r - \frac{5}{6}\pi r^3$	4	
	(b)	$r = 4$ m, $h = 4$ m	4	
9	(a)	Proved	4	
	(b)	$x = 26.6°, 135°, 206.6°, 315°$	6	
10	(a)	$x_B = 1.767$	3	
	(b)	$\int_0^{x_B} y\,dx = P - Q$	2	
	©	$P - Q = \dfrac{2-\sqrt{13}}{3} \approx -0.535$	4	
	(d)	$Q > P$	1	
11	(a)	$m_L = \tan 45° = 1$	1	
	(b)(i)	$y = -x + 10;\ D(5,5)$	6	
	(b)(ii)	14 units^2	3	
12	(a)	$x(x^2-1)^2 = 0$	3	
	(b)	$x = -1,\ 0,\ 1$	1	
	(c)(i)	$(-1,5),\ (1,5)$	2	
	(c)(ii)	$(0,2)$ is a minimum	2	

DIFFERENTIATION AND INTEGRATION

Question 1

YOUR SCORE ___ / 4

(a) Differentiate $x^2 \sin x$ with respect to x. [2]

1. Use the product rule with $u = x^2$ and $v = \sin x$. M1
$$\frac{d}{dx}(x^2 \sin x) = x^2 \cdot \cos x + \sin x \cdot 2x$$

2. $= 2x \sin x + x^2 \cos x$ A1

ANSWER (A) $2x \sin x + x^2 \cos x$

(b) Find $\int \frac{3}{2x+1}\, dx$. [2]

1. The integral of $\frac{1}{ax+b}$ is $\frac{1}{a}\ln(ax+b)$, so a $\ln(2x+1)$ term appears. M1
$$\int \frac{3}{2x+1}\, dx = 3 \cdot \frac{1}{2}\ln(2x+1) + c$$

2. $= \frac{3}{2}\ln(2x+1) + c$ A1

ANSWER (B) $\frac{3}{2}\ln(2x+1) + c$

Where students lose marks

- Forgetting the $+c$ on an indefinite integral costs the A1.
- Writing $3\ln(2x+1)$: the $\frac{1}{2}$ from the chain rule is missing.

Will I still get the mark?

YES **I wrote $\frac{3}{2}\ln|2x+1| + c$ with modulus signs.**
Modulus signs are correct and fully accepted. They are not required at O-Level.

YES **I left (a) as $x^2 \cos x + 2x \sin x$.**
The order of terms does not matter. Full marks.

PARTLY **I forgot $+c$ in (b).**
You keep the M1 for the $\ln(2x+1)$ form, but the final A1 needs the constant of integration.

SURDS (NO CALCULATOR)

Question 2

YOUR SCORE ___ / 6

A calculator must not be used in this question.

(a(i)) Given that $x = \dfrac{-5+\sqrt{57}}{2}$ satisfies $x^2 + 5x = k$, find the constant k. [2]

1. Substitute and expand. Alternatively, notice that this is the quadratic formula for $x^2 + 5x - k = 0$. **M1**

$$x^2 = \frac{82 - 10\sqrt{57}}{4}$$

$$5x = \frac{-50 + 10\sqrt{57}}{4}$$

$$x^2 + 5x = \frac{32}{4}$$

2. $k = \dfrac{32}{4} = 8$ **A1**

ANSWER (A(I)) $k = 8$

(a(ii)) State the other solution of $x^2 + 5x = k$ in surd form. [1]

1. The roots of $x^2 + 5x - 8 = 0$ are $\frac{-5\pm\sqrt{57}}{2}$, so take the other sign. **B1**

$$x = \frac{-5 - \sqrt{57}}{2}$$

ANSWER (A(II)) $x = \dfrac{-5 - \sqrt{57}}{2}$

(b) Find the gradient of the line through $(1, \sqrt{2})$ and $(\sqrt{2}, 4\sqrt{2})$ in the form $a + b\sqrt{2}$. [3]

1. $m = \dfrac{4\sqrt{2} - \sqrt{2}}{\sqrt{2} - 1} = \dfrac{3\sqrt{2}}{\sqrt{2} - 1}$ **M1**

2. Rationalise by multiplying by the conjugate $\sqrt{2} + 1$. **M1**

$$m = \frac{3\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{6 + 3\sqrt{2}}{2 - 1}$$

3. $m = 6 + 3\sqrt{2}$ **A1**

ANSWER (B) $6 + 3\sqrt{2}$

Where students lose marks

- Using a calculator to "find" k earns nothing. The working must show exact surd manipulation.
- Putting the y-difference over the wrong x-difference flips the sign of the denominator.

Will I still get the mark?

YES **For (a) (i) I compared with the quadratic formula: $b^2 - 4ac = 25 + 4k = 57$, so $k = 8$.**
That is a valid and neat method. Equating the discriminant earns the M1, and $k = 8$ earns the A1.

YES **I used the sum of roots $= -5$ to get the other root in (a) (ii).**
$-5 - \frac{-5+\sqrt{57}}{2} = \frac{-5-\sqrt{57}}{2}$. Any correct route earns the B1.

PARTLY **My final gradient was $\frac{3\sqrt{2}}{\sqrt{2}-1}$.**
That earns the first M1 only. The question asks for the form $a + b\sqrt{2}$, so you must rationalise.

DISCRIMINANT: LINE MEETS CURVE

Question 3

YOUR SCORE ___ / 6

The curve $y = (k-10)x^2 + (k-8)x - (2k+3)$ meets the line $y = 2x + k$ at two distinct points. Find the set of possible values of k. [6]

1. Equate the curve and the line. M1
$(k-10)x^2 + (k-8)x - (2k+3) = 2x + k$

2. Collect everything on one side. M1
$(k-10)x^2 + (k-10)x - (3k+3) = 0$

3. Two distinct points of intersection, so $b^2 - 4ac > 0$. M1
$(k-10)^2 + 4(k-10)(3k+3) > 0$

4. Factor out $(k-10)$ rather than expanding everything. A1
$(k-10)(k-10+12k+12)$
$= (k-10)(13k+2) > 0$

5. The critical values are $k = -\frac{2}{13}$ and $k = 10$. The parabola in k opens upwards, so the expression is positive outside the roots. M1

6. $\{k : k < -\frac{2}{13} \text{ or } k > 10\}$ A1

ANSWER $k < -\frac{2}{13}$ or $k > 10$

Where students lose marks

- The equation is only quadratic when $k \neq 10$. The answer already excludes $k = 10$, so no extra step is needed, but $k = 10$ must not be included.
- Using $\geqslant$ gives "at least one point", not two distinct points.

Will I still get the mark?

YES **I expanded fully and got $13k^2 - 128k - 20 > 0$, then solved it.**
That is the same inequality, since $(k-10)(13k+2) = 13k^2 - 128k - 20$. Full marks with the correct final set.

PARTLY **I wrote $-\frac{2}{13} < k < 10$.**
Method marks are kept, but the region is the wrong side of the critical values, so both final A marks are usually lost.

PARTLY **I wrote the answer as $k < -0.154$ or $k > 10$.**
The exact value $-\frac{2}{13}$ is expected. A rounded decimal is likely to lose the final A1.

EXPONENTIAL GROWTH WITH LOGARITHMS

Question 4

YOUR SCORE ___ / 7

An email is forwarded so that each new person sends it to two new people. Level 1 is the original sender, so level n has 2^{n-1} new people.

(a) The process stops as soon as a level has over $1\,000\,000$ new people. Find that level. [3]

1. Levels 1, 2, 3 have 1, 2, 4 people, so level n has 2^{n-1}.
   $2^{n-1} > 1\,000\,000$ — B1

2. Take logarithms of both sides. — M1
   $n - 1 > \dfrac{\lg 10^6}{\lg 2} = 19.93\ldots$

3. The smallest integer is $n - 1 = 20$. Check: $2^{19} = 524\,288$ and $2^{20} = 1\,048\,576$. — A1
   $n = 21$

ANSWER (A) Level 21

(b) Find the first level where the new people at that level and the level before it add up to more than 8000. [4]

1. $2^{n-1} + 2^{n-2} > 8000$ — B1

2. Factor out 2^{n-2}. — M1
   $2^{n-2}(2 + 1) > 8000$
   $2^{n-2} > 2666.\overline{6}$

3. $n - 2 > \dfrac{\lg 2666.\overline{6}}{\lg 2} = 11.38\ldots$ — M1

4. So $n - 2 = 12$. Check: levels 13 and 14 have $4096 + 8192 = 12\,288 > 8000$, while levels 12 and 13 have $2048 + 4096 = 6144$. — A1
   $n = 14$

ANSWER (B) Level 14

Where students lose marks

- Using 2^n instead of 2^{n-1} shifts every answer by one level.
- Rounding 19.93 down to 19 instead of up. You need the first whole number that exceeds the bound.

Will I still get the mark?

PARTLY **I listed powers of 2 and counted instead of using logs.**
A correct answer from a clear, systematic table usually earns the B1 and A1. The M1 for logarithms is likely to be withheld, because the question is set on exponentials and logarithms.

PARTLY **I got level 20 in (a) by using $2^n > 10^6$.**
The logarithm method earns the M1, but the B1 for the correct model and the final A1 are lost.

NO **For (b) I used the sum of a GP from level 1.**
The question only adds the current level and the one before it, not every level from the start. A cumulative sum answers a different question.

LINE MEETS CURVE: SIMULTANEOUS EQUATIONS

Question 5

YOUR SCORE ___ / 6

Find the coordinates of the points where the curve $\frac{x}{y}+\frac{2}{x}=1$ meets the line $y=2x-3$. [6]

1. Multiply the curve through by xy. **M1**
   $x^2+2y=xy$
2. Substitute $y=2x-3$. **M1**
   $x^2+2(2x-3)=x(2x-3)$
3. $x^2+4x-6=2x^2-3x$ **A1**
   $x^2-7x+6=0$
4. $(x-1)(x-6)=0$ **M1**
5. $x=1$ or $x=6$ **A1**
6. Find y from the line. Neither point has $x=0$ or $y=0$, so both are valid. **A1**
   $(1,-1)$ and $(6,9)$

ANSWER $(1,-1)$ and $(6,9)$

Where students lose marks

- Multiplying by x only, or by y only, and leaving a fraction behind leads to messy algebra and sign slips.
- Giving only the x-values. The question asks for coordinates.

Will I still get the mark?

YES **I made x the subject of the line and substituted for x instead.**
Eliminating x gives a quadratic in y with roots $y=-1$ and $y=9$. It is valid and earns the same marks.

PARTLY **I found $x=1,6$ but forgot to find y.**
You can get up to 5 of the 6 marks. The final A1 needs both coordinate pairs.

COMPLETING THE SQUARE

Question 6

YOUR SCORE ____ / 8

(a) Express $5x^2 - 10x + k$ as $A(x + B)^2 + C$, where C is in terms of k. [3]

1. Factor 5 out of the x terms.
$5x^2 - 10x + k = 5(x^2 - 2x) + k$ — B1

2. $= 5\left[(x - 1)^2 - 1\right] + k$ — B1

3. $= 5(x - 1)^2 + k - 5$ — B1

ANSWER (A) $A = 5,\ B = -1,\ C = k - 5$

(b(i)) State the coordinates of the turning point of $y = 5x^2 - 10x + k$ in terms of k. [2]

1. Read the turning point from the completed square: $(x - 1)^2 = 0$ when $x = 1$.
$(1,\ k - 5)$ — B1 B1

ANSWER (B(I)) $(1,\ k - 5)$

(b(ii)) Explain why this turning point is a minimum. [1]

1. Since $(x - 1)^2 \geqslant 0$ for all x, we have $y = 5(x - 1)^2 + k - 5 \geqslant k - 5$, so the smallest value of y occurs at the turning point. Equivalently, the coefficient of x^2 is $5 > 0$, so the curve opens upwards. — B1

ANSWER (B(II)) $y \geqslant k - 5$ for all x

(b(iii)) The curve is never below the line $y = 3$. Find the set of possible values of k. [2]

1. The lowest point of the curve must be on or above $y = 3$.
$k - 5 \geqslant 3$ — M1

2. $\{k : k \geqslant 8\}$ — A1

ANSWER (B(III)) $k \geqslant 8$

Where students lose marks

- "Never below" allows the curve to touch $y = 3$, so the inequality is $\geqslant$, not $>$.
- Writing $B = 1$. The form is $(x + B)^2$, so $(x - 1)^2$ means $B = -1$.

Will I still get the mark?

PARTLY **I wrote $k > 8$ in (b)(iii).**
M1 for $k - 5$ compared with 3, but the A1 is lost. At $k = 8$ the curve just touches $y = 3$, which is still "never below".

YES **In (b)(ii) I used $\frac{d^2y}{dx^2} = 10 > 0$.**
A correct second-derivative argument is a valid explanation for the B1.

YES **In (b)(iii) I used $b^2 - 4ac \leqslant 0$ on $5x^2 - 10x + k - 3$.**
$100 - 20(k - 3) \leqslant 0$ gives $k \geqslant 8$. It is a valid method for both marks.

EQUATION OF A CIRCLE

Question 7

YOUR SCORE ___ / 7

A dartboard has two concentric circles centred at $(2, 3)$. A dart lands on the outer circle at $(5, -4)$.

(a) Find the equation of the outer circle. [3]

1. The radius is the distance from the centre $(2, 3)$ to $(5, -4)$. **M1**
$r^2 = (5-2)^2 + (-4-3)^2$
$= 9 + 49 = 58$

2. Use $(x-a)^2 + (y-b)^2 = r^2$. **B1** **A1**
$(x-2)^2 + (y-3)^2 = 58$

ANSWER (A) $(x-2)^2 + (y-3)^2 = 58$

(b) The inner circle is $x^2 + y^2 - px - qy - 23 = 0$. Find p, q and the radius of the inner circle. [4]

1. Its centre is $\left(\frac{p}{2}, \frac{q}{2}\right)$, which must be $(2, 3)$ because the circles are concentric. **M1**
$\frac{p}{2} = 2, \quad \frac{q}{2} = 3$

2. $p = 4, \quad q = 6$ **A1**

3. $r^2 = 2^2 + 3^2 + 23 = 36$ **M1**

4. $r = 6$ **A1**

ANSWER (B) $p = 4,\ q = 6,\ r = 6$

Where students lose marks

- Sign of the centre: $x^2 + y^2 - px - qy + c = 0$ has centre $\left(+\frac{p}{2}, +\frac{q}{2}\right)$.
- For the inner radius, $r^2 = g^2 + f^2 - c$ with $c = -23$, so add 23 rather than subtract it.

Will I still get the mark?

YES **I left the answer to (a) as $x^2 + y^2 - 4x - 6y - 45 = 0$.**
The expanded form is the same circle. Full marks.

PARTLY **I wrote $r = \sqrt{58}$ but never wrote the equation.**
M1 for the radius, but the question asks for the equation of the circle.

NO **I got $r^2 = 4 + 9 - 23$.**
That gives a negative r^2, which should have been a warning sign. The final M1 needs $g^2 + f^2 - c$ with the correct sign of c.

MAXIMUM VOLUME (DIFFERENTIATION)

Question 8

YOUR SCORE ____ / 8

A tank is an open-topped cylinder (radius r m, height h m) with a hemisphere of the same radius on top and a circular base. The total metal area is 80π m².

(a) Show that the volume is $V = 40\pi r - \frac{5}{6}\pi r^3$. [4]

1. The surfaces are the hemisphere ($2\pi r^2$), the curved side of the cylinder ($2\pi rh$) and the base (πr^2). **B1**
$$2\pi r^2 + 2\pi rh + \pi r^2 = 80\pi$$
$$3r^2 + 2rh = 80$$

2. $$h = \frac{80 - 3r^2}{2r}$$ **M1**

3. The volume is the cylinder plus the hemisphere. **M1**
$$V = \pi r^2 h + \tfrac{2}{3}\pi r^3$$
$$= \pi r^2 \cdot \frac{80 - 3r^2}{2r} + \tfrac{2}{3}\pi r^3$$

4. $$V = 40\pi r - \tfrac{3}{2}\pi r^3 + \tfrac{2}{3}\pi r^3$$
$$= 40\pi r - \tfrac{5}{6}\pi r^3 \text{ (shown)}$$ **A1**

ANSWER (A) $V = 40\pi r - \frac{5}{6}\pi r^3$

(b) The radius can vary. Find the radius and the cylinder height that give the largest volume (no need to prove it is a maximum). [4]

1. $$\frac{dV}{dr} = 40\pi - \frac{5}{2}\pi r^2$$ **M1**

2. Stationary when $\frac{dV}{dr} = 0$. **M1**
$$\frac{5}{2}\pi r^2 = 40\pi \Rightarrow r^2 = 16$$

3. Reject $r = -4$ because a length is positive. **A1**
$$r = 4$$

4. $$h = \frac{80 - 3(16)}{2(4)} = \frac{32}{8} = 4$$ **A1**

ANSWER (B) $r = 4$ m, $h = 4$ m

Where students lose marks

- Including a top circle for the cylinder. It is open at the upper end, where the hemisphere sits.
- Using the full sphere area $4\pi r^2$ for the hemisphere instead of $2\pi r^2$.
- For a "show that", every line must lead clearly to the given result. Jumping straight to the answer loses the A1.

Will I still get the mark?

PARTLY **I wrote $r = \pm 4$ and did not reject -4.**
Most markers still award M1 M1, but the A1 for r needs the negative root rejected or ignored with $r = 4$ stated.

PARTLY **I gave the maximum volume instead of the height.**
Marks for $r = 4$ are kept. The question asks for the height of the cylinder, $h = 4$, so the last A1 is lost.

YES **I justified that it is a maximum even though the question said not to.**
Extra correct work is not penalised. It just costs time.

TRIGONOMETRIC IDENTITY, HENCE SOLVE

Question 9

YOUR SCORE ___ / 10

(a) Prove that $\tan x(\sin x + \cos x)(\operatorname{cosec} x + \sec x) = (\tan x + 1)^2$. [4]

1. Start from the left-hand side and write the reciprocal functions in terms of $\sin x$ and $\cos x$. **M1**

$$\text{LHS} = \tan x(\sin x + \cos x) \times \left(\frac{1}{\sin x} + \frac{1}{\cos x}\right)$$

2. $$= \tan x(\sin x + \cos x) \cdot \frac{\cos x + \sin x}{\sin x \cos x}$$ **M1**

3. Use $\tan x = \frac{\sin x}{\cos x}$, which cancels the $\sin x$. **M1**

$$= \frac{\sin x}{\cos x} \cdot \frac{(\sin x + \cos x)^2}{\sin x \cos x}$$
$$= \frac{(\sin x + \cos x)^2}{\cos^2 x}$$

4. $$= \left(\frac{\sin x}{\cos x} + 1\right)^2$$
$$= (\tan x + 1)^2 = \text{RHS}$$ **A1**

ANSWER (A) Proved

(b) Hence solve $(\sin x + \cos x)(\operatorname{cosec} x + \sec x) = 3(\tan x + 1)$ for $0° < x < 360°$. [6]

1. From (a), the left-hand side equals $\frac{(\tan x+1)^2}{\tan x}$. **M1**

$$\frac{(\tan x + 1)^2}{\tan x} = 3(\tan x + 1)$$
$$(\tan x + 1)^2 = 3\tan x(\tan x + 1)$$

2. Do not cancel $(\tan x + 1)$, which would lose two solutions. Factorise instead. **M1**

$$(\tan x + 1)(\tan x + 1 - 3\tan x) = 0$$
$$(\tan x + 1)(1 - 2\tan x) = 0$$

3. $\tan x = -1$ or $\tan x = \frac{1}{2}$ **A1**

4. Basic angles are $45°$ and $\tan^{-1}\frac{1}{2} = 26.565°$. Tangent is negative in quadrants II and IV and positive in I and III. **M1**

5. $\tan x = \frac{1}{2}$: $x = 26.6°,\ 206.6°$ **A1**

6. $\tan x = -1$: $x = 135°,\ 315°$ **A1**

ANSWER (B) $x = 26.6°, 135°, 206.6°, 315°$

Where students lose marks

- Cancelling $(\tan x + 1)$ on both sides throws away $135°$ and $315°$. This is the trap in this question.
- Angles must be to 1 decimal place: $26.6°$ and $206.6°$.
- In a proof, work on one side only. Do not cross-multiply both sides together.

Will I still get the mark?

PARTLY **I cancelled $\tan x + 1$ and only got $26.6°$ and $206.6°$.**
You lose the factorising M1, the A1 for both tan values and the A1 for $135°, 315°$. Expect about 3 out of 6.

YES **Are $135°$ and $315°$ really valid? $\sin x + \cos x = 0$ there.**
Both sides equal 0, and $\sin x$, $\cos x$ and $\tan x$ are all defined and non-zero, so both angles satisfy the original equation.

YES **For (a) I worked from the right-hand side to the left.**
Either direction is fine, as long as you transform one side into the other in a clear chain.

PARTLY **I wrote $26.57°$ and $206.57°$.**
Angles in degrees should be given to 1 decimal place. Over-specifying can cost the accuracy mark, although some markers allow it once.

AREA UNDER A TRIGONOMETRIC CURVE

Question 10

YOUR SCORE ___ / 10

The curve $y = 2\sin 3x + 3\cos 3x$ crosses the x-axis at A and then B. Region P (above the axis) runs from $x = 0$ to A, and region Q (below the axis) runs from A to B.

(a) Find the x-coordinate of B in radians, correct to 4 significant figures. [3]

1. $2\sin 3x + 3\cos 3x = 0$
   $\tan 3x = -\frac{3}{2}$ — M1
2. The basic angle is $\tan^{-1} 1.5 = 0.98279$. Tangent is negative in quadrants II and IV, so A uses $3x = \pi - 0.98279$ and B uses the next solution, $3x = 2\pi - 0.98279$. — M1
   $3x = 2\pi - 0.98279 = 5.30039$
3. $x = 1.767$ (4 s.f.) — A1

ANSWER (A) $x_B = 1.767$

(b) Explain why the x-coordinate of A is not needed to find $P - Q$. [2]

1. Area above the axis gives a positive integral and area below gives a negative integral, so $\int_A^B y\,dx = -Q$. — B1
2. So one integral from 0 to B already equals $P + (-Q) = P - Q$, and the limit at A is never used. — B1
   $$\int_0^{x_B} y\,dx = \int_0^{x_A} y\,dx + \int_{x_A}^{x_B} y\,dx$$
   $$= P - Q$$

ANSWER (B) $\int_0^{x_B} y\,dx = P - Q$

(c) Find the value of $P - Q$. [4]

1. $\int (2\sin 3x + 3\cos 3x)\,dx$ — M1 A1
   $= -\frac{2}{3}\cos 3x + \sin 3x$
2. At B, $3x$ is in quadrant IV with $\tan 3x = -\frac{3}{2}$, so $\cos 3x = \frac{2}{\sqrt{13}}$ and $\sin 3x = -\frac{3}{\sqrt{13}}$. — M1
   $P - Q = \left(-\frac{4}{3\sqrt{13}} - \frac{3}{\sqrt{13}}\right)$
   $- \left(-\frac{2}{3}\right)$
3. $P - Q = \dfrac{2 - \sqrt{13}}{3} \approx -0.535$ — A1

ANSWER (C) $P - Q = \dfrac{2 - \sqrt{13}}{3} \approx -0.535$

(d) What does the answer to © say about the sizes of P and Q? [1]

1. $P - Q < 0$, so Q is larger than P, by about 0.535 square units. **B1**

ANSWER (D) $Q > P$

Where students lose marks

- Set the calculator to radians. A degree-mode value of $\tan^{-1} 1.5$ ruins (a) and ©.
- Taking $3x = \pi - 0.98279$ gives A, not B.
- Integrating $\sin 3x$ to $\cos 3x$ without the $-\frac{1}{3}$ factor.

ALTERNATIVE METHOD · PART (A)

Use the R-formula for (a)

1. $2\sin 3x + 3\cos 3x$
$= \sqrt{13}\sin(3x + \alpha)$,
$\alpha = \tan^{-1}\frac{3}{2} = 0.98279$ **M1**

2. The second zero after $x = 0$ is where $3x + \alpha = 2\pi$.
$3x = 2\pi - 0.98279$
$x = 1.767$ **M1** **A1**

Will I still get the mark?

YES **I wrote $P - Q = -0.535$ without the exact form.**
A correct 3 s.f. decimal is accepted for the final A1.

YES **I worked out P and Q separately and subtracted.**
$P = 1.869$ and $Q = 2.404$ give $P - Q = -0.535$. It is longer but valid for ©. For (b), you still need the explanation.

YES **For (d) I wrote "Q is bigger".**
That earns the B1. Saying it is because $P - Q$ is negative makes it clearer.

PARTLY **I gave $x_B = 1.77$.**
The question asks for 4 significant figures, so the A1 is lost.

COORDINATE GEOMETRY: TRAPEZIUM AND SYMMETRY

Question 11

YOUR SCORE ___ / 10

Trapezium $ABCD$ has line of symmetry L. AB is parallel to the x-axis and CD is parallel to the y-axis. In the diagram, A is top-left, B is top-right and D is bottom-right.

(a) Explain why the gradient of L must be 1. [1]

1. Reflecting in L maps the horizontal side AB onto the vertical side DC. A mirror line that swaps horizontal and vertical directions makes $45°$ with both, and here it slopes upwards, so its gradient is $\tan 45° = 1$. B1

ANSWER (A) $m_L = \tan 45° = 1$

(b(i)) L is $y = x + 8$ and A is $(-3, 13)$. Find the equation of AD and hence the coordinates of D. [6]

1. D is the mirror image of A in L, so AD is perpendicular to L.
   $m_{AD} = -1$ B1
2. $y - 13 = -(x + 3)$ M1
3. $y = -x + 10$ A1
4. Where AD meets L is the midpoint of AD.
   $x + 8 = -x + 10 \Rightarrow x = 1,\ y = 9$ M1
5. $M = (1, 9)$ A1
6. $D = (2 \cdot 1 - (-3),\ 2 \cdot 9 - 13) = (5, 5)$ A1

ANSWER (B(I)) $y = -x + 10;\ D(5, 5)$

(b(ii)) Given $AB = 2$ units, find the area of the trapezium. [3]

1. B is 2 units to the right of A, so $B = (-1, 13)$. By symmetry, C is the image of B in L, or equivalently $DC = 2$ upwards from D.
   $B(-1, 13), \quad C(5, 7)$ B1 B1
2. Use the shoelace formula, taking the vertices in order A, B, C, D.
   $\frac{1}{2}\begin{vmatrix} -3 & -1 & 5 & 5 & -3 \\ 13 & 13 & 7 & 5 & 13 \end{vmatrix}$
3. Down-right products: $-39 - 7 + 25 + 65 = 44$. Up-right products: $-13 + 65 + 35 - 15 = 72$.
   $\text{Area} = \frac{1}{2}|44 - 72| = 14 \text{ units}^2$ A1

ANSWER (B(II)) 14 units^2

Where students lose marks

- The parallel sides are AD and BC (both perpendicular to L), not AB and CD.
- Listing the shoelace vertices out of order gives a wrong area.

ALTERNATIVE METHOD · PART (B(II))

Trapezium formula with perpendicular parallel sides

1 $AD = \sqrt{8^2 + 8^2} = 8\sqrt{2}$
$BC = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ — B1

2 The distance between $y = -x + 10$ and $y = -x + 12$ is $\frac{2}{\sqrt{2}} = \sqrt{2}$. — M1 A1
$\text{Area} = \frac{1}{2}(8\sqrt{2} + 6\sqrt{2})\sqrt{2}$
$= 14$

Will I still get the mark?

PARTLY — **I reflected A in $y = x + 8$ directly using $(x, y) \mapsto (y - 8, x + 8)$.**
You get the right D, but the question says "find the equation of AD and hence", so marks for the equation of AD are lost if you never write it.

YES — **I used $\frac{1}{2}(AD + BC) \times$ height with $8\sqrt{2}$, $6\sqrt{2}$ and height $\sqrt{2}$.**
$\frac{1}{2}(8\sqrt{2} + 6\sqrt{2})(\sqrt{2}) = 14$. A fully valid method.

STATIONARY POINTS OF INFLEXION

Question 12

YOUR SCORE ____ / 8

The curve is $y = 3x^6 - 9x^4 + 9x^2 + 2$.

(a) Show that the x-coordinates of the stationary points satisfy $x(x^2-1)^2 = 0$. [3]

1. $\frac{dy}{dx} = 18x^5 - 36x^3 + 18x$ — M1
2. $= 18x(x^4 - 2x^2 + 1) = 18x(x^2-1)^2$ — M1
3. At stationary points $\frac{dy}{dx} = 0$, and dividing by 18 gives the result. $x(x^2-1)^2 = 0$ (shown) — A1

ANSWER (A) $x(x^2-1)^2 = 0$

(b) Solve the equation to find the x-coordinates of the three stationary points. [1]

1. $x = -1,\ 0,\ 1$ — B1

ANSWER (B) $x = -1,\ 0,\ 1$

(c(i)) The curve has one turning point and two stationary points of inflexion. Find the inflexion points and justify that they are points of inflexion. [2]

1. Substitute $x = \pm 1$: $y = 3 - 9 + 9 + 2 = 5$. $(1, 5)$ and $(-1, 5)$ — B1
2. Since $(x^2-1)^2 \geqslant 0$, the sign of $\frac{dy}{dx}$ is the sign of x. Near $x = 1$ it is positive on both sides (e.g. $x = 0.9$ and 1.1), and near $x = -1$ it is negative on both sides. The gradient does not change sign, so both are stationary points of inflexion. — B1

ANSWER (C(I)) $(-1, 5),\ (1, 5)$

(c(ii)) Determine the nature of the turning point. [2]

1. At $x = 0$, $\frac{dy}{dx}$ changes from negative ($x = -0.1$) to positive ($x = 0.1$). — M1
2. $(0, 2)$ is a minimum point — A1

ANSWER (C(II)) $(0, 2)$ is a minimum

Where students lose marks

- Using $\frac{d^2y}{dx^2}$ at $x = \pm 1$: it equals 0 there, which proves nothing. Use the first-derivative sign test.
- Testing values too far from ± 1, such as $x = 0$ or $x = 2$, can cross another stationary point.

Will I still get the mark?

YES

In © (ii) I used $\frac{d^2y}{dx^2} = 18 > 0$ at $x = 0$.

$\frac{d^2y}{dx^2} = 90x^4 - 108x^2 + 18$, which is 18 at $x = 0$, so the point is a minimum. That is a valid method for both marks.

NO

For © (i) I said "$\frac{d^2y}{dx^2} = 0$, so it is a point of inflexion".

A zero second derivative is inconclusive. The B1 needs evidence that $\frac{dy}{dx}$ does not change sign.

YES

I only found $(1, 5)$ and said the other one follows by symmetry.

The question tells you the curve is symmetrical about the y-axis, so stating $(-1, 5)$ by symmetry is fine.

PAPER 2 IS NEXT

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